

LOCAL RISK PRONENESS IN ANALYTICALLY  
APPROXIMATED UTILITY FUNCTIONS UNDER  
MONOTONICALLY DECREASING PREFERENCES

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**Abstract**

We discuss an analytical arctan form to approximate decreasing utilities based on several nodes of its graphics elicited in interval form. We demonstrate the process on two types of nodes originating from different subjective elicitation approaches. Our focus is also on the local risk attitude estimator, which in the case of decreasing preferences gets interpreted as local risk proneness vs the local risk aversion for increasing preferences.

**Key words:** von Neumann–Morgenstern utility, subjective elicitation, risk attitude

**Introduction.** The objective of decision making theory (DMT) is to rank the alternatives in the set  $L$  according to the preferences of the decision maker (DM). The consequences of his/her choice depend not only on the choice itself, but also on a random component. The consequences may be one-dimensional, but usually are described as a multi-dimensional vector with attributes that describe important aspects of the decision for the DM. Each alternative is in fact a random vector of consequences with a given probability distribution. As a special case, there may be alternatives with certain consequences. The set of consequences  $X$  is the set of all possible consequences of the alternatives in  $L$ .

Let  $\vec{x}_{best}$  and  $\vec{x}_{worst}$  be the best and the worst consequences in  $X$ . The reference lottery (RL) is a hypothetical alternative, which gives either  $\vec{x}_{best}$  with probability  $p$  or  $\vec{x}_{worst}$  with probability  $(1 - p)$ . The RL is denoted as  $\langle \vec{x}_{best}(p)\vec{x}_{worst} \rangle$ .

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The DM can rank the consequences in  $X$  in descending order of the values of the utility function  $u(\vec{x})$ . The latter is the probability in that reference lottery, which is indifferent to  $\vec{x}$ : if  $\langle \vec{x}_{best}(p)\vec{x}_{worst} \rangle \sim \vec{x}$  then  $u(\vec{x}) = p$ . Here,  $u(\vec{x}_{best}) = 1$ ,  $u(\vec{x}_{worst}) = 0$ . In fact the utility solves the problem of ranking the alternatives since according to the utility existence theorem [1] the expected utility  $E(u|q)$  of a given alternative  $q$  is the probability in that reference lottery, which is indifferent to the alternative:  $\langle \vec{x}_{best} E(u|q)\vec{x}_{worst} \rangle \sim q, \forall q \in L$ .

It is evident that the rational DM should rank the alternatives according to their expected utility [2,3]. This is the most common interpretation of the utility function, a.k.a. utility with probabilistic interpretation (UPI). The utility uniqueness theorem claims that each positive affine transformation of an UPI leads to strategically equivalent utilities [4].

Let us assume that the consequences are 1-D quantities with monotonic preference and the respective set  $X$  is continuous set from  $x_{min}$  to  $x_{max}$ . Our aim is to build a continuous utility function  $u(\cdot)$  in the domain  $[x_{min}; x_{max}]$ . For example, such situation occurs when the consequences are purely monetary. The shape of the utility function over the elements of  $X$  will reflect the risk attitude of the DM, i.e. whether he/she is risk prone, risk averse or risk neutral. There are several quantitative measures of the risk attitude proposed in literature [5,6]. The most common one is the function

$$(1) \quad r(x) = -u''(x)/u'(x)$$

(a.k.a. local risk aversion), which depends solely on the utility function, but not on the alternatives [1].

The significance of  $r(\cdot)$  is based on a theorem [1,7] stating that if  $u_1(\cdot)$  and  $u_2(\cdot)$  are utility functions of two DMs such that over a given set  $X$ ,  $r_1(x)$  is greater than  $r_2(x)$ , then for each lottery with consequences in  $X$  it is true that  $(I_p)_1 > (I_p)_2$ . Here,  $(I_p)_i$ ,  $i = 1, 2$ , is the risk premium for the  $i$ -th DM. It is the quantity  $x$  that the DM is willing to pay not to carry the risk of the lottery  $q$ . It means that the  $i$ -th DM will sell the lottery for  $(I_p)_i$  less than its expected value  $E(x|q)$ . Using  $r(\cdot)$  it is possible to define the risk attitude of the DM depending on the range of consequences. Empirical proof suggests that, for increasing preferences over profit, most people are decreasingly risk prone when losses increase, decreasingly risk averse when profits increase, and shift quickly from explicit risk proneness to explicit risk aversion around the zero value. A typical increasing utility function and its corresponding local risk aversion are given in Fig. 1 [8]. Another work [9] derives practically the same curve without curve fitting, but utilizing eight parameters with biophysical meaning related to human neural activities.

Constructing the utility function directly over the consequences is rarely seen in such a pure form since  $\vec{x}$  is usually a multi-dimensional vector. However, in

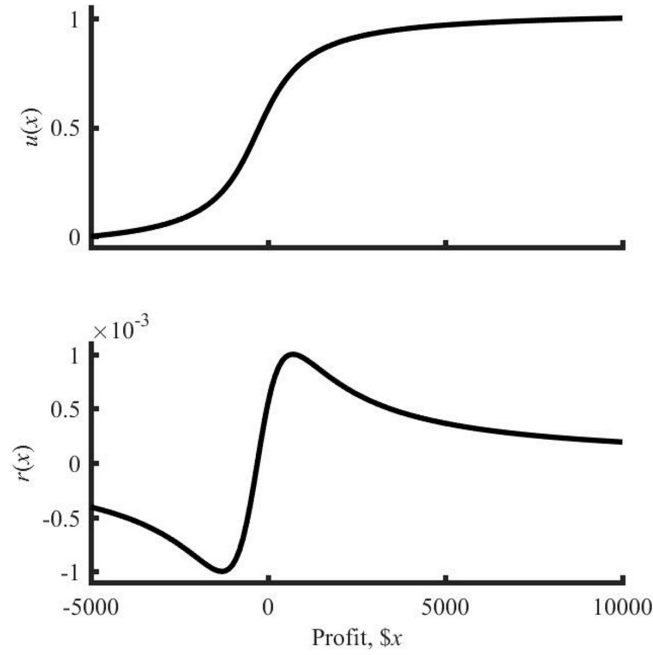


Fig. 1. a) typical  $u(\cdot)$  over profits from  $-\$5000$  to  $\$10\,000$ ;  
b) local risk aversion  $r(\cdot)$  of  $u(\cdot)$  from a)

many cases [7] the utility function may exist over each attribute of the vector (under certain conditions of independence of preference), and the overall utility function depends on those attribute utility functions as well as on some scaling constants. In practice, each problem contains some sort of monetary component where the preferences are monotonic.

Different analytical forms for increasing preference were proposed. Some of them [10] use the Harington's function of desirability  $u(x) \sim_p e^{-e^{-b-ax}}$  (we use  $\sim_p$  to denote proportionality). This dependence generates a function with a typical local risk aversion for profits. The form  $u(x) \sim_p \arctan(ax - ax_0)$  is suggested for both profits and losses [11]. Another suggestion [7] is that the utility analytical form should depend on the risk attitude. For risk neutrality, constant risk aversion, decreasing risk aversion, constant risk proneness and decreasing risk proneness the utility should be, respectively,  $u(x) \sim_p x$ ,  $u(x) \sim_p -e^{-cx}$  for  $c > 0$ ,  $u(x) \sim_p \ln(x)$ ,  $u(x) \sim_p x^2$ , and  $u(x) \sim_p (x + b)^a$  for  $b > -x$  and  $a > 1$ . CLEMEN [12] proposes  $u(x) \sim_p 1 - e^{-x/R}$ , which is suitable for constant risk aversion  $R$ .

In this paper, we will discuss the construction of continuous utility function under monotonically decreasing preferences.

**Materials and methods.** Let  $x_{min} = x_{best} = \inf(X)$  and  $x_{max} = x_{worst} = \sup(X)$ . We need to construct the one-dimensional  $u(\cdot)$  in the interval  $[x_{best}; x_{worst}]$

provided that the DM's preferences decrease monotonically in  $X$ . This requires a dialog with the DM [4] to obtain estimates of several nodes of  $u(\cdot)$ , after which we can either perform linear interpolation on the estimated nodes or nonlinear approximation with a given analytical form.

The methods for subjective elicitation of utility function nodes divide into two groups. In the first one, we search for the utilities of pre-selected consequences (e.g. probability equivalence (PE) [12] and lottery equivalence (LE) [13] methods). In the second group, we look for the consequences corresponding to pre-selected utilities (e.g. certainty equivalence (CE) [14] and uncertain equivalence (UE) [15] methods). It is possible to find adaptations of the most common utility elicitation methods to the case of strictly decreasing preferences [16].

Let the DM elicit in an interval form  $z - 2$  inner quantiles from  $u(\cdot)$  with coordinates  $(x_{u_l}, u_l)$  for  $l = 2, 3, \dots, z - 1$ , where  $x_{u_l}$  and  $u_l$  are the utility quantile and the index of the utility quantile, respectively. The end nodes are known:  $(x_{u_1}, u_1) = (x_{best}, 1) = (x_{min}, 1)$  and  $(x_{u_z}, u_z) = (x_{worst}, 0) = (x_{max}, 0)$ . If we use the first group of methods, then the nodes will have uncertainty interval along the ordinate. They are described as

$$(2) \quad \left\{ \begin{array}{l} \{(x_l; \hat{u}_l^d; \hat{u}_l^u) \mid l = 1, 2, \dots, z\}, \text{ where} \\ x_{min} = x_1 < x_2 < \dots < x_z = x_{max}, \\ 1 = \hat{u}_1^d \geq \hat{u}_2^d \geq \dots \geq \hat{u}_z^d = 0, \\ 1 = \hat{u}_1^u \geq \hat{u}_2^u \geq \dots \geq \hat{u}_z^u = 0, \\ \hat{u}_l^d < \hat{u}_l^u \text{ for } l = 2, 3, \dots, z - 1 \end{array} \right.$$

and named  $u$ -nodes.

If we use the second group of methods, then the nodes will have uncertainty interval along the abscissa. They are described as

$$(3) \quad \left\{ \begin{array}{l} \{(\hat{x}_{u_l}^d; \hat{x}_{u_l}^u; u_l) \mid l = 1, 2, \dots, z\}, \text{ where} \\ x_{min} = \hat{x}_{u_1}^d \leq \hat{x}_{u_2}^d \leq \dots \leq \hat{x}_{u_z}^d = x_{max}, \\ x_{min} = \hat{x}_{u_1}^d = \hat{x}_{u_1}^u \leq \hat{x}_{u_2}^u \leq \dots \leq \hat{x}_{u_z}^u = \hat{x}_{u_z}^d = x_{max}, \\ \hat{x}_{u_1}^d < \hat{x}_{u_1}^u \text{ for } l = 2, 3, \dots, z - 1, \\ 1 = u_1 > u_2 > \dots > u_z = 0 \end{array} \right.$$

and named  $x$ -nodes.

In this paper, we will explore the utility approximation in the case of decreasing preferences with the analytical function:

$$(4) \quad u(x, \vec{p}) = \frac{\arctan[a(x_{max} - x_0)] - \arctan[a(x - x_0)]}{\arctan[a(x_{max} - x_0)] - \arctan[a(x_{min} - x_0)]}.$$

The local risk attitude estimator  $r(x)$  that corresponds to the utility (4) is:

$$(5) \quad r(x, \vec{p}) = -\frac{u''(x)}{u'(x)} = \frac{2a^2(x - x_0)}{1 + a^2(x - x_0)^2}.$$

The vector of unknown parameters  $\vec{p} = (a, x_0)$  is two-dimensional, and can be identified by weighted least square (WLS) minimisation in the two-dimensional set  $\Pi$  defined as:

$$(6) \quad \Pi = \{(a, x_0) \mid a \in (0, \infty) \text{ and } x_0 \in (-\infty, \infty)\}.$$

The local risk attitude estimator (5) of the analytical utility function for strictly decreasing preferences (4) coincides with the local risk aversion of the utility function for strictly increasing preferences [11]. This has its logical explanation. The definition of  $r(\cdot)$  in (1) is designed for strictly increasing preferences over  $X$ , where the consequences and the risk premiums are interpreted as profits. Since the preferences over  $X$  are now strictly decreasing, the consequences and the risk premiums are interpreted as losses. Then the positive risk premium (positive  $r(x)$ ) means that the DM is willing to give away the risk of the alternative and “pays” for that by giving away losses. In other words, the DM sells the risk of the alternative and behaves as a risk prone individual. In the same way, the negative risk premium (negative  $r(x)$ ) means that the DM is willing to give away the risk of the alternative in exchange of a guaranteed (certain) loss, which is higher than the expected loss of the alternative. In other words, the DM buys certainty and behaves as a risk averse individual. We can see that the local risk attitude estimator for the case of strictly decreasing preferences can be interpreted as “local risk proneness”.

The proposed form (4) is strictly decreasing in the domain  $[x_{min}; x_{max}]$ :

$$(7) \quad u(x_{low}, \vec{p}) > u(x_{high}, \vec{p}) \Leftrightarrow x_{low} < x_{high} \text{ for } \{x_{low}, x_{high}\} \subset [x_{min}; x_{max}] \text{ and } \vec{p} \in \Pi.$$

It is zero at  $x_{max}$ , and one at  $x_{min}$ , which makes it UPI:

$$(8) \quad u(x_{worst}, \vec{p}) = u(x_{max}, \vec{p}) = 0, \quad u(x_{best}, \vec{p}) = u(x_{min}, \vec{p}) = 1 \text{ for } \vec{p} \in \Pi.$$

However, the main advantage of the arctan approximation (4) is that its local risk attitude estimator (5) coincides with the empirically proven one shown in Fig. 1. This is why we expect that in most cases, this analytical form would give approximation of high quality. In reality, the approximation (4) uses external prior information about the form of the utility function, so the model node value would depend on the uncertainty intervals of all empirically elicited nodes. This fact allows additional smoothing of the function  $u(\cdot)$  and reduces the measurement error.

In the case of  $u$ -nodes the end nodes are error-free. Then the unknown parameters of (4) may be identified using WLS [17], where the deviation of the model from the best subjective point estimate at a given node is weighted by the

width of the uncertainty interval of the index of the utility quantile. A goodness-of-fit measure of (4) with the data (2) is:

$$(9) \quad \chi_u^2(\vec{p}) = \sum_{l=2}^{z-1} [e_{u,i}(\vec{p})]^2 = \sum_{l=2}^{z-1} \left( \frac{2u(x_l, \vec{p}) - (\hat{u}_l^d + \hat{u}_l^u)}{2(\hat{u}_l^u - \hat{u}_l^d)} \right)^2.$$

The optimal parameters  $\vec{p}_{opt}$  may be identified with 2-dimensional minimization of  $\chi_u^2$  on  $\vec{p}$ :

$$(10) \quad \vec{p}_{opt} = (a_{opt}; x_{0,opt}) = \arg \left\{ \min_{\vec{p}} \{ \chi_u^2(\vec{p}) \} \right\}.$$

From (7) it follows that there exists an inverse of (4). Luckily, it is also an analytical function, which speeds up the calculations:

$$(11) \quad x(u, \vec{p}) = \frac{\tan\{(1-u) \cdot \arctan[a(x_{max} - x_0)] + u \cdot \arctan[a(x_{min} - x_0)]\}}{a} + x_0.$$

Similar to (4), the form (11) would be strictly decreasing and fixed at its ends:

$$(12) \quad x(u_{low}, \vec{p}) > x(u_{high}, \vec{p}) \Leftrightarrow u_{low} < u_{high} \text{ for } \{u_{low}, u_{high}\} \subset [0; 1] \text{ and } \vec{p} \in \Pi,$$

$$(13) \quad x(0, \vec{p}) = x_{max}, \quad x(1, \vec{p}) = x_{min} \text{ for } \vec{p} \in \Pi.$$

In the case of  $x$ -nodes the end nodes are error-free. Then the unknown parameters of 11 may be identified using a WLS [17], where the deviation of the model from the best subjective point estimate in a given node is weighted by the width of the uncertainty interval of the utility quantile. A goodness-of-fit measure of 11 with the data (3) is:

$$(14) \quad \chi_x^2(\vec{p}) = \sum_{l=2}^{z-1} [e_{x,i}(\vec{p})]^2 = \sum_{l=2}^{z-1} \left( \frac{2x(u_l, \vec{p}) - (\hat{x}_{u_l}^d + \hat{x}_{u_l}^u)}{2(\hat{x}_{u_l}^u - \hat{x}_{u_l}^d)} \right)^2.$$

The optimum parameters  $\vec{p}_{opt}$  may be found using 2-dimensional minimization of  $\chi_x^2$  on  $\vec{p}$ :

$$(15) \quad \vec{p}_{opt} = (a_{opt}; x_{0,opt}) = \arg \left\{ \min_{\vec{p}} \{ \chi_x^2(\vec{p}) \} \right\}.$$

**Results.** We present two examples to demonstrate the ideas in the previous sections.

*Example 1. Time to complete home repair.* We discuss a one-dimensional continuous random variable  $X$ , which is the time in days for a home repair in the

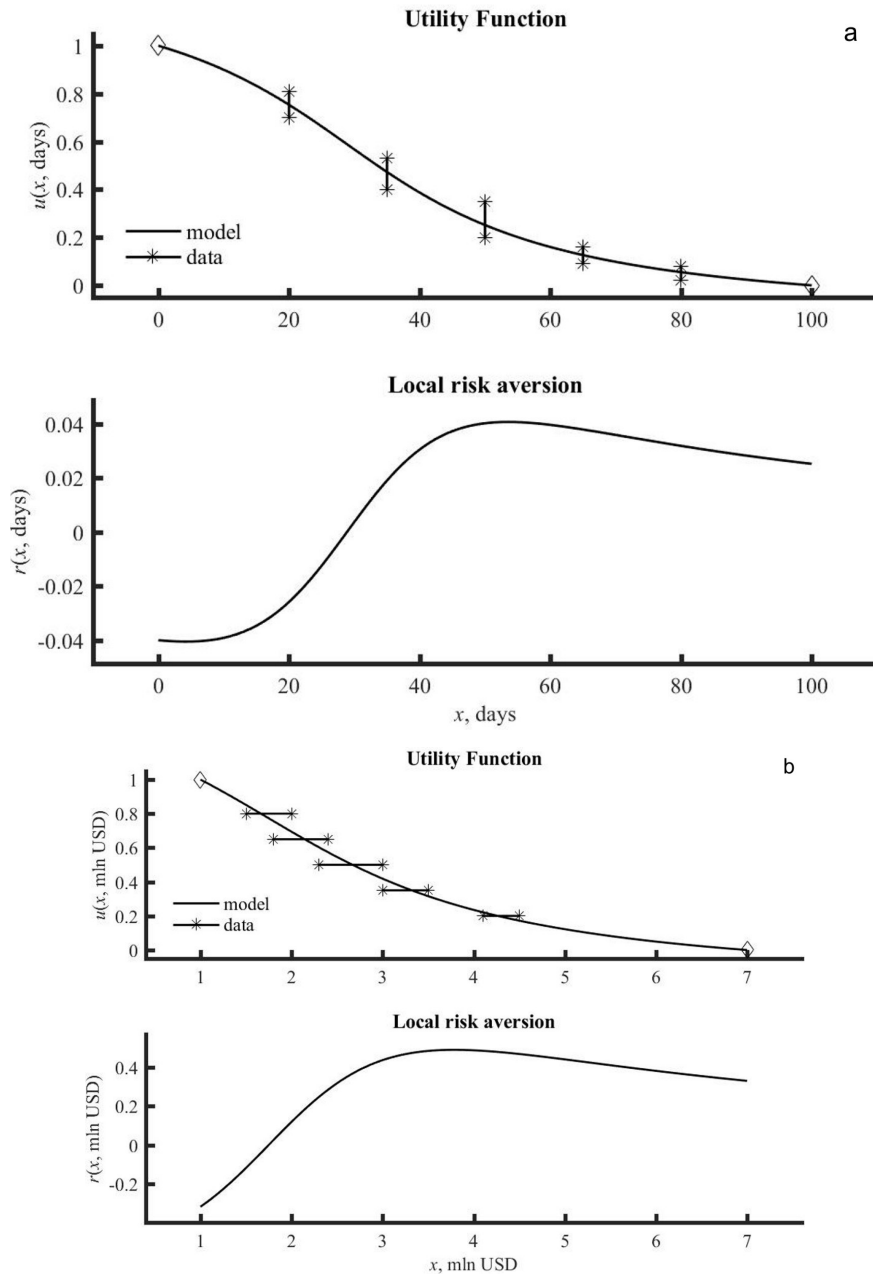


Fig. 2. a) arctan-approximated utilities for Example 1;  
b) arctan-approximated utilities for Example 2

interval  $[0; 100]$  when the repair is planned for 30 days. Using the LE method we select five inner utility quantiles  $x_{u_2} = 20$ ,  $x_{u_3} = 35$ ,  $x_{u_4} = 50$ ,  $x_{u_5} = 65$ ,  $x_{u_6} = 80$  and find their corresponding indices:  $\hat{u}_2 \in [\hat{u}_2^d; \hat{u}_2^u] \equiv [0.70; 0.81]$ ,  $\hat{u}_3 \in$

$[\hat{u}_3^d; \hat{u}_3^u] \equiv [0.40; 0.53]$ ,  $\hat{u}_4 \in [\hat{u}_4^d; \hat{u}_4^u] \equiv [0.20; 0.35]$ ,  $\hat{u}_5 \in [\hat{u}_5^d; \hat{u}_5^u] \equiv [0.09; 0.16]$ ,  $\hat{u}_6 \in [\hat{u}_6^d; \hat{u}_6^u] \equiv [0.02; 0.08]$ .

Based on those elicited nodes and using (10) we find the optimum parameters of the analytical utility function:  $a_{opt} = 4.06 \times 10^{-2}$ ,  $x_{0,opt} = 28.9$ ,  $\chi_{opt}^2 = 0.0356$ . The graphics of the resulting utility function and its corresponding “local risk proneness” function are given in Fig. 2a. It is evident that the arctan-approximation well describes the elicited nodes and the model passes by the uncertainty intervals of the  $u$ -nodes. All local errors from (9) are less than 50%:  $e_{u,2} = -0.014$ ,  $e_{u,3} = 0.063$ ,  $e_{u,4} = -0.155$ ,  $e_{u,5} = 0.019$ ,  $e_{u,6} = 0.085$ .

**Example 2. Annual expenses for a hospital.** We discuss a one-dimensional continuous random variable  $X$ , which is the annual expenses of a hospital. The DM has strictly decreasing preferences in the interval  $[1; 7]$  (in million USD), when 2.7 mln USD is budgeted. We must construct the one-dimensional utility function.

We select five inner indices of the utility quantiles:  $u_2 = 0.8$ ,  $u_3 = 0.65$ ,  $u_4 = 0.5$ ,  $u_5 = 0.35$ ,  $u_6 = 0.20$  and find their corresponding utility quantiles. Using UE we obtain:  $\hat{x}_{u_2} \in [\hat{x}_{u_2}^d; \hat{x}_{u_2}^u] \equiv [1.5; 2]$ ,  $\hat{x}_{u_3} \in [\hat{x}_{u_3}^d; \hat{x}_{u_3}^u] \equiv [1.8; 2.4]$ ,  $\hat{x}_{u_4} \in [\hat{x}_{u_4}^d; \hat{x}_{u_4}^u] \equiv [2.3; 3]$ ,  $\hat{x}_{u_5} \in [\hat{x}_{u_5}^d; \hat{x}_{u_5}^u] \equiv [3; 3.5]$ ,  $\hat{x}_{u_6} \in [\hat{x}_{u_6}^d; \hat{x}_{u_6}^u] \equiv [4.1; 4.5]$ .

Based on those elicited nodes and using (15) we find the optimum parameters of the analytical utility function:  $a_{opt} = 0.491$ ,  $x_{0,opt} = 1.748$ ,  $\chi_{opt}^2 = 0.0634$ . The graphics of the resulting utility function and its corresponding “local risk proneness” function are given in Fig. 2b. It is evident that the arctan-approximation well describes the elicited nodes and the model passes by the uncertainty intervals of the  $x$ -nodes. All local errors from (14) are less than 50%:  $e_{x,2} = -0.173$ ,  $e_{x,3} = 0.071$ ,  $e_{x,4} = 0.026$ ,  $e_{x,5} = 0.139$ ,  $e_{x,6} = -0.092$ .

**Discussion.** As a whole, we need to treat the quantity (1) differently and name it differently depending on whether preferences over  $X$  are decreasing or increasing. The popular name of (1) – “local risk aversion” – only makes sense in the case of increasing preferences. Should we decide to use only the term ‘local risk aversion’, then the sign of (1) needs to change to plus for decreasing preference. However, the latter would lead to confusions in the interpretation.

The better quality of prior information that the local risk proneness contains allows obtaining the same quality of approximation of  $u(x)$  using fewer elicited nodes (which in turn saves a lot of time), similar to another empirical proof [10].

The analytical form (4) almost always gives excellent results in approximating utilities. However, each time we need to check if the utilized prior information for  $r(\cdot)$  is applicable to that DM, i.e. if the approximated curve passes through the elicited uncertainty intervals of the nodes (the relative errors  $e_{x,i}(\vec{p}_{opt})$  or  $e_{u,i}(\vec{p}_{opt})$  should belong to  $[-0.5; 0.5]$ ). If not, then this information is inapplicable for that particular DM and deteriorates the analysis. The DM should then revert to linear interpolation on the centroids of the uncertainty intervals of the nodes. So, the proposed approach is safe to use.



The numerical optimization problems (10) and (15) are solved with the function `LSQNONLIN` of MATLAB<sup>®</sup> using the trust-region reflective least squares algorithm with analytical Jacobian [18]. We utilized a set of original functions written in MATLAB<sup>®</sup> to perform all calculations and visualizations in the examples. These are available free upon request from the authors.

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